where N_e is the electrical power that drives the sound column.

All these equations were developed in Herbert Petzoldt, *Electroakustics,* vol. IV (in German).

ON THE STABILITY OF A VOLTAGE-CONTROLLED FILTER/RESONATOR

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The operational amplifier filter circuit reported by Colin[1] is an interesting application of analog computing theory. However, Colin's claim for *unconditional* stability is not justified by the scant information concerning his electronic multipliers.

The theoretical problem occurs when the "constant" K_2 of his Fig. 2 is set to a very low value to obtain a high Q band-pass filter. For this condition, the value of σ_0 in his pole-zero plot of Fig. 3 approaches zero and the poles lie very close to the $j\omega$ axis, although supposedly in the (stable) left half of the s plane. Should the poles move to the $j\omega$ axis, the impulse (meaning the Dirac delta function) response becomes a sinusoid, and the shock excitation by switching transients or power supply induced noise will result in a single tone output or tone generation rather than filtering.

It is doubtful that the poles would move to just the $j\omega$ axis as σ_0 approaches zero, and this is why the circuit cannot be called unconditionally stable. The well-known problem in analog computer circuitry is phase shift in potentiometers, multipliers, and the like. In Colin's circuit, it is most essential for K_3 and K_4 to be electronically controllable to set the resonance frequency of the system. In the absence of any details about the electronic multipliers which establish K_3 and K_4, one must assume they are typical of the better computer grade multipliers and exhibit small but finite phase shift, especially at the upper end of the audio spectrum. This small amount of phase shift moves the system poles into the right-hand side of the s plane which is a most unstable region. Indeed, noise excitation of the system will cause oscillation to start, and the magnitude will increase until some nonlinearity in the system reaches a saturation value, probably when the output of the operational amplifiers approaches the power supply potential.

Lest you think this is an improbable set of circumstances, a favorite way for professors to discourage analog computer salesmen is to patch up Colin's Fig. 2 on their demonstration machine with K_2 set to zero by leaving it out. The constants K_3 and K_4 are adjusted to make the resonance frequency 1 kHz, and a small step function is applied to the input. The machines promptly take off, and the oscillation builds up the point where the overload alarm is howling within just a few seconds. The problem is well known to be the *phase shift* in the helical potentiometers used to set K_3 and K_4. Unless Colin's patented electronic multipliers are an order of magnitude better than those available for high-performance analog computers, he cannot claim unconditional stability for his circuit.

[1] D. P. Colin, "Electrical Design and Musical Applications of an Unconditionally Stable Combination Voltage Controlled Filter/Resonator," *J. Audio Eng. Soc.,* vol. 19, pp. 923-927 (Dec. 1971).

Reply by Mr. Colin

Dr. Ashley's thorough knowledge of poles and zeros has led him to the correct conclusion that excess phase shift in either of the two loops shown in my Fig. 2 will move the two poles toward the $j\omega$ axis, causing self-oscillation. In addition, conventional analog multipliers are typically limited to an output voltage dynamic range of 40 dB, or 100 to 1. For this reason alone, their use was prohibited in my filter design, since it was desired to cover at least the audio range from 16 Hz to 16 kHz. What this all boils down to is the crux of the whole design problem: the need for an accurately controlled gain function with a dynamic range of at least 60 dB (1000 to 1), and a frequency response to well above 16 kHz so that phase shift within the audio range is not excessive. I designed a variable-gain integrator which, of course, achieves the same function as a multiplier followed by an integrator, and I have filed for a patent to be assigned to Tonus, Inc. Regrettably, I am not free to disclose actual circuit information at this time, but the variable-gain integrator achieves a dynamic range of 100 dB (10^5 to 1) typically, and 80 dB (10^4 to 1) minimum. The small amount of phase shift (above the desired 90° shift of the integrator, of course) is compensated for by a simple trimmable lead–lag network to the extent that the filter will operate at a Q of 500 without drifting into oscillation, at any frequency from 16 Hz to 16 kHz. This is an exceedingly high Q for musical purposes; the impulse response of the filter at this Q and a center frequency of 1 kHz is a 1 kHz sinusoid that takes over 1 second to decay to 60 dB below the initial amplitude. The filters actually do this, and this is what I mean by unconditional stability; *not* the ability to oscillate at exactly zero damping, that is, without growing or decaying in amplitude, which is what Dr. Ashley proposed attempting in the analog computer circuit with K_2 set to zero. In my filter, K_2 is an exponential function of control voltage ($K_2 = 2^{-V_c}$), therefore it never reaches zero.

The filter does perform according to specification, and is now in use by many satisfied musicians, composers, and engineers. In addition, I have used the filter for spectral analysis of audio signals, including the response of a calibrated microphone to a loudspeaker driven with pink noise. The latter plot agreed well with the swept sine-wave frequency response of the same loudspeaker/microphone/room response. I would be happy to demonstrate the filter to anyone who expresses interest by writing or calling me either at Tonus, Inc., 45 Kenneth Street, Newton Highlands, Mass. 02161, phone (617) 969-0810; or at home, 72 Pleasant Street, Wakefield, Mass. 01880, phone (617) 246-0677.

In conclusion, I am grateful for the opportunity to reply to Dr. Ashley, whom I admire for his excellent work in the field of loudspeaker design and measurement.